2.1 USE OF SIMPSON'S RULES FOR NUMERICAL INTEGRATION

: For calculation of areas, first and second moment of area, volume etc.

: assumed that the bounding curves are portions of parabolas.

Simpson's first rule, Simpson's second rule, Simpson's third rule, Trapezoidal rule.

SIMPSONS FIRST RULE

: All ordinates must be equally spaced

: The number of ordinates must be odd

: $A = 1/3\ h\ (Y1 + 4Y_2 - Y_3)$ for 3 ordinates.

: $A - 1/3\ h\ (Y1 + 4Y_2 + 2Y_3 + 4Y_4 + Y_5)$ for 5 ordinates through computing of multipliers.

: $A - 1/3\ h\ (½\ Y_1 + 2\ Y_2 + 1\ ½\ Y3 + 4\ Y_4 + 2\ Y_5 - 4\ Y_6 + 1\ ½\ Y_7 - 2Y_8 + ½\ Y_9)$ for half stations at the ends.

: h = distance between two full ordinates.

: h = (total length) / (total number of full stations minus one) ---------- for water planes.

: h– (total depth) / (total number of full water lines minus one) -------- for transverse sections.

SIMPSON'S SECOND RULE

: All ordinates must be equally spaced
: The number of ordinates must 3n+1 where n=1, 2, 3.....etc.
: $A = 3/8.h\ (Y_1 + 3Y_2 + 3Y_3 + Y_4)$ for 4 ordinates.
: $A + 3/8.h\ (Y_1 + 3Y_2 + 3Y_3 + 2Y_4 + 3Y_5 + 3Y_6 + Y_7)$
For 7 ordinates through compounding of multipliers.

: h= Distance between two full ordinates.

: h= (total length) / (total number of full stations minus one) ----- For water planes.
: h = (total depth) / (total number of full water lines minus one) ---- for transverse sections.

SIMPSON'S THIRD RULE (5, 8,-1 RULE)

: All ordinates must be equally spaced

: The number of ordinates must be three

: The area calculated will be between the 5 and 8 multiplied ordinates.

: $A = h/12\ (5Y_1 + 8Y_2 - Y_3)$

: h = Distance between two full ordinates.

TRAPEZOIDAL RULE

: All ordinates must be equally spaced

: can be used for any number of ordinates.

: $A = \frac{1}{2} h (Y_1 + 2Y_2 + 2Y_3 + \text{----------} Yn)$ for n ordinates.

: h= distance between two full ordinates.

: h = (total length) / (total number of full stations minus one) ------- for water planes

: h = (total depth) / (total number of full water lines minus one) -------- for transverse sections.

: The results not so accurate as that of Simpson's rule.

Tchebycheff's rules

This system differs from the Simpson rules in that the ordinates are not equi-spaced. The spacing is such that the ordinates do not require internal multipliers but can be directly added. The spacing is such that the average of the measured ordinates is the mean ordinate of the figure. The end ordinates of the figure are not used. The area is then obtained by adding together the lengths of the measured ordinates dividing by the number of ordinates and finally multiplying by the length of the figure for which the area is desired. The spacing given in table are fractions of half length of base and is measured from the middle of the base.

Length of base = L; No. of ordinates used = N; Area = L/N (sum of measured ordinates)

No. of ordinates used	Position of ordinates from the middle of the base in fractions of the half length					
N	amidships	1	2	3	4	5
2		0·5773				
3	0	0·7071				
4		0·1876	0·7947			
5	0	0·3745	0·8325			
6		0·2666	0·4225	0·8662		
7	0	0·3239	0·5297	0·8839		
8		0·1026	0·4062	0·5938	0·8974	
9	0	0·1679	0·5288	0·6010	0·9116	
10		0·0838	0·3127	0·5000	0·6873	0·9162

Fig.2.22 Tchebycheff's rule

Example

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Tutorial No.1

1. Obtain the Simpson's multipliers which should be used to integrate the half breadths of the stations:

   Stn.No. 0, ¼, ½, ¾, 1, 1.5, 2, 2.5, 3, 4, 5

   Assume that the outside multiplier is ⅓xhx2 where h is the common interval between two full sations.

2. Find the area of the waterplane of a ship 120m long using Simpson's 1st and 2nd rule.

Stn.No.	0	1	2	3	4	5	6
½ B	0	8.2	. 10.1	9.3	7.8	3.9	0

3. Calculate the area of the waterplane for which the half breadths are given below.

St No.	0	0.5	1	3	5	7	9	9.5	10
½ B	.76	1.52	2.12	3.21	3.25	3.25	1.56	0.72	0

   The length of the waterplane is 42m.

4. The half areas of the waterplanes of a ship are given below.

WL(m)	0	0.3	0.6	0.9	1.2	1.5	1.8
½Area	0	41.1	56.9	68.3	75.3	79.8	83.5

   Calculate the volume of water displaced by the ship if the draught is 1.8m.

5. The half breadths of a midship section starting from the top are 7.48, 7.45, 7.27, 6.9, 5.76, 4.42, and 2.68m respectively.
   The common interval between the consecutive half-breadths is 1m between the first and the fifth half-breadths and half that value between the fifth and the seventh.
   Calculate the area of the midship section.

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Tutorial No.2

1. The half-breadths of the waterplane of a ship 100m long are as follows: .

St. No.	0	0.5	1	2	3	4	5	6	7	7.5	8
½B	0	2.7	3.6	5	5	5	5	4.1	2.9	1.8	0

Calculate the area of the waterplane using:

a) Simpson's first rule
b) Trapezoidal rule

2. The half-breadths of a midship section of a ship are given below:

WL.No.	0	0.5	1	2	3	4
½B	6.4	8.8	10.9	12.8	13.6	14.2

Using Simpson's rule and all the half breadths, calculate the area of the midship sections at 0.5Wl, 1WL,2WL,3WL and 4WL respectively given that the vertical distance between WL No.0 and WL No.4 is 8 m.

3. The half-breadths spaced at equal interval of a ship's waterplane of length 70m are 0, 11.2, 15, 15, 15, 15, 10 and 0 meters respectively. Calculate the area of the waterplane using:

a) Combination of Simpson's first and second rule.
b) Combination of Simpson's 2nd and 5,8,-1 rule
c) Trapezoidal rule

4. The half-breadths starting from the top of a transverse section of a ship with waterline spacing 1m are 6.0, 5.9, 5.8, 5.6, 5.4, 4.8 and 0 meters respectively.

Calculate the area of transverse section using:

a) Combination of Simpson's 1st and 2nd rule
b) Combination of Simpson's 1st and 5,8,-1 rule
c) Trapezoidal rule

5. The half-breadths of equal spacing of a ship's waterplane of length 100m are 0, 5, 9, 13, 15, 15, 15, 15, 12, 7 and 0 meters respectively.

Calculate the area of the waterplane.

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NAVAL ARCHITECTURE I
Tutorial No.3

1. Compare the areas given by Simpson's Rules and Trapezoidal Rule fof the portions of the curve defined below:
   a) between ordinates 1 and 4
   b) between ordinates 1 and 2

Ord.No.	1	2	3	4
Ord(m)	39.0	19.0	12.6	10.0

The common interval between two ordinates is 10m.

2. Show that the area for a curvilinear figure with 6 ordinates, Y_1, Y_2, Y_3, Y_4, Y_5, and Y_6 with the common interval between the ordinates of h is given by:

   $$\text{Area} = (25/24)h(0.4Y_1+Y_2+Y_3+Y_4+Y_5+0.4Y_6)$$

   Hint:Use Simpson's 2nd rule on Y_2 to Y_5 and apply 5,8,-1 at the end segments.

3. The waterline of a ship is 70 m long. Its half ordinates which are equally spaced are given below. Calculate the area of waterplane using Simpson's rule and the Trapezoidal rule.

Ord. No	1	2	3	4	5	6	7	7.5	8
½B	0.0	3.1	6.0	8.4	10	10.1	8.6	6.4	0.0

4. Assuming the truth of the 5,8,-1 rule for finding area between two consecutive ordinates, prove the truth of the rule known as Simpson's 1st rule.

5. A curvilinear figure has ordinates 3m apart of length 9.7, 10.0, and 13.3 m respectively. Find:
   a) The area between the first and the second ordinates
   b) The area between second and third ordinates.
   c) check the addition of these results by finding the area of the whole figure by Simpson's 1st rule.

6. The half-breadths in m of the waterplane of a ship 88m long are as follows:

St. No	0	0.5	1	2	3	4	5	6	7	7.5	8
½B	0	2.7	3.6	4.7	4.8	4.8	4.8	4.1	2.9	1.8	0

Find the area of the waterplane.

7. A semi-circle has a diameter of 20m. Find the area of the semi-circle using Tchebycheff's five ordinate rule. Compare the answer with that obtained by formula for semi-circle.

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Tutorial No.4

1. The half ordinates of a deck plan are 0.0, 0.38, 1.68, 3.12, 4.11, 4.50, 4.42, 3.81, 2.74, and 1.07m. The length of the deck plan is 39.01m.
   a) Find the area of the deck plan
   b) Find the area of the deck plan between the last two ordinates.

2. The half-breadths of a midship section are 6.8, 6.77, 6.61, 6.28, 5.24, 4.02, and 2.44m. The common interval between the consecutive ordinates is 0.91m between the first and the fifth ordinate and half that value between the fifth and the seventh. Calculate the midship section coefficient using Simpson's and Trapezoidal rule.

3. The semi ordinates of a load waterplane of a vessel are 0.06, 1.10, 2.26, 3.05, 3.35, 3.26, 2.83, 1.68 and 0.61 m respectively and they are 4.57m apart.
   a) What is the waterplane area coefficient of the load waterplane?
   b) What is the tonnes per cm immersion of this waterplane?

4. The areas of waterplane respectively in sq.m are 200, 200, 160, 125, and 30. The common interval between waterplanes is 0.45m. Find the displacement of the vessel:
   a) in cubic meters.
   b) in Mega newtons in fresh water
   c) in tonnes force in sea water.

   Assume the mass density of fresh water and sea water to be 1000 and 1025 kg per cubic meter resp. and g=9.8^1 m/sec^2

5. The transverse sections spaced at 3.3 m apart on a vessel have the following areas: 1,5,6,7,4 and 2 m^2. Find the volume of displacement and the displacement in seawater in MN and in tonne force.

6. A ship has the following particulars:
   Length : 152.4 m
   Breadth : 17.37 m
   Draft : 6.86 m
   Displ : 10017 tonne force
   Immersed midship area: 120 m^2

   Find the block coefficient, prismatic coefficient and midship section coefficient.

7. A ship 176.79m L.B.P has a beam of 20.12m and a draft of 6.4m. If the prismatic coefficient is 0.66, the area of waterplane 2694.25 m^2 and the displacement 14427.2 tonnef, Find:
   a) Block coefficient
   b) Waterplane coefficient
   c) Midship section coefficient.

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Tutorial No.5

1. The length, beam and mean draft of a ship are respectively 115, 15.65, and 7.15m. Its midship section coefficient is 0.921 and its block coefficient is 0.665. Find:

   a) displacement in tonnef and newtons in sea water.
   b) area of immersed midship section.
   c) prismatic coefficient of displacement.

2. Two similar right circular cones are joined at their bases. The composite body floats so that both apexes are in the water surface. Calculate:

   a) midship section coefficient
   b) prismatic coefficient
   c) waterplane area coefficient.

3. A quadrant of 16m radius is divided by means of ordinates parallel to one radius and at the following distances: 4, 8, 10, 12, 13, and 15m. The lengths of these ordinates are 15.49, 13.86, 12.49, 10.58, 9.33, 7.75, and 5.57m. Find:

   a) the area to two decimal places by formula
   b) the area using only ordinates 4 m apart by Simpson's rule
   c) the area using also the half ordinates.
   d) the area using all the ordinates given.
   e) the area using all the ordinates except 12.49.

4. Calculate the area of a circle of diameter 40m using:

   a) the standard formula for area of circle
   b) using 5 ordinates with Simpson's first rule
   c) using Tchebycheff's rules with 5 ordinates.

5. A tank is 8m deep throughout its length and 20m long and its top flat and horizontal. The sections forward in the middle and at the aft end are all triangular, apex down and the widths of the triangles at the tank top are respectively 15, 12 and 8m.
   Draw the calibration curve for the tank in tonnef of oil fuel against depth and the capacity from the calibration curve when the depth of oil is 5.5m. Given: SG of oil fuel is 0.9. Only 5 points of the curve need be calculated.

6. The shape of a steel plate of S.G 7.85 is given below.

Ord No	1	2	3	4	5	6
Breadth(m)	5.3	5	4.5	3.8	3.0	1.4

The thickness of the steel plate is 15mm. Calculate the steel weight in Newtons and in tonnef.

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NAVAL ARCHITECTURE I
Tutorial No.6

1. The half breadths of a waterline of a ship of length 110m are given below:

Stn No	0	1	2	3	4	5	6	7	8	9	10
½B(m)	0	4.6	6.0	6.1	6.1	6.1	6.1	6.0	5.2	3.7	0

Find the position of the LCF of waterplane relative to:

a) Station number 0
b) Station number 5
c) Station number 10

2. The half breadths of a waterline of a tanker are:

St.No.	1	2	3	4	5	6	7	8	9	10
½B(m)	0	2.7	4.4	5.8	6.0	6.0	6.0	5.4	3.6	0

The length of waterplane is 81m. Find the following:

1) the area of waterplane
2) the TPCM in tonnef(assume sea water)
3) the position of L.C.F from stn. No.1

3. The half breadths of a waterplane of a barge are:

St. No.	1	2	3	4	5	6	7	8
½.B(m)	4.5	8.0	9.0	9.0	9.0	8.6	7.8	6.0

The length of the waterplane is 91m. Calculate:

a) the area of waterplane
b) the position of L.C.F from St. No. 1

4. The half breadths of a waterplane of a ship are:

St. No.	0	0.5	1	2	3	4	5	6	7	7.5	8
½B(m)	0	4.2	6.9	9.4	10	10	10	9.1	6.5	3.9	0

The length of the waterplane is 120m. Assume that the midship is located at St. No.4. Calculate the following:

a) the area of waterplane coefficient
b) the position of L.C.F from midship.

5. The half breadths of a waterplane of a supply vessel are:

St.No.	1	2	3	4	5	6
½.B(m)	0	4.8	6.0	6.0	4.2	0

The length of waterplane is 60m. Calculate the TPCM in sea water and the position of LCF relative to St. No.1.

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NAVAL ARCHITECTURE I
Tutorial No.7

1. Areas of waterplane 2.5m apart, of a tanker are:

Waterplane	0	0.5	1	2	3	4	5
Area(m^2)	200	700	1700	3100	3800	4000	4010

Calculate the volume of displacement and position of VCB above the keel.

2. A ship 137m long floats at a waterline at which the transverse sectional areas, at equal spacing from aft are:
0,165, 349, 471, 526, 561, 593, 625, 617, 253, 0 m^2 respectively. Calculate the displacement in tonneforce and Meganewtons and the position of L.C.B from amidships.

3. The cross sectional area of a cargo hold, at evenly spaced sections starting from the aft end are 0, 9.0, 11.9, 13.9, 15.2, 16.1 and 16.7 tonnef per cm immersion respectively. Calculate the displacement in tonnef when floating in sea water and the centre of buoyancy above the keel.

4. Calculate the displacement in tonnef and Meganewtons and find the LCB and VCB of the ship at 3.5m WL. The half breadths in meters are given below.

Waterline	0.5m	1.0m	1.5m	2.5m	3.5m
Stations					
0(AP)	0.06	0.21	0.55	1.28	1.95
1	1.43	2.59	3.60	4.39	4.66
2	2.26	3.72	4.60	4.94	5.0
3	1.31	2.38	3.38	4.27	4.56
4	0.06	0.15	0.30	0.91	1.83

Assume ship to be floating in sea water. The length of the ship is 32m.

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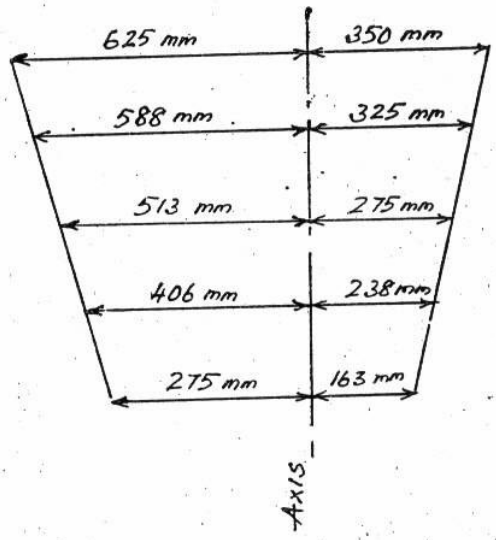
NAVAL ARCHITECTURE I
Tutorial No.8

1. The ordinates of a rudder abaft its axis spaced at 300mm apart and commencing from the top are 1.53, 1.47, 1.41, 1.29, 1.14, 0.99, and 0.81m respectively. Find the position of the centroid relative to the top ordinate and relative to the rudder axis.

2. A spade rudder is shown below with its respective ordinates spaced at 250mm. Calculate the total area of the rudder and find the centroid of the rudder relative to the top ordinate and relative to the rudder axis.



3. A piece of stealer plate of length 14m and thickness 12mm has the following ordinates beginning from the after end:
2.4, 2.4, 2.3, 2.2, 2.1, 1.7, 1.5 and 1m respectively. Calculate the weight of the stealer plate in Meganewtons and the position of the centroid relative to the aft end and the straight edge of the plate.

4. The ordinates of a skeg of length 10m fitted to the bottom of a flat top barge starting from the after end are 2.5, 1.8, 1.3, 0.4, and 0m respectively. Calculate the area of the skeg and the position of the centroid of area relative to the after end and relative to the keel.

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NAVAL ARCHITECTURE I
Tutorial No.9

1. A box type barge has a length of 30m, breadth of 7m and depth of 3m. Calculate at 2.5m draft the second moment of area of waterplane of the barge about its axis through its centroid parallel to
   a) the length
   b) the breadth
   What is the transverse GM of the barge at 2.5m draft if KG=3m? Is the barge stable ?

2. The half ordinates of a ship at 7m draft, starting from the aft are 0.2, 7.4, 8.7, 9.0, 9.1, 9.2, 9.1, 8.6, 7.8, 5.1 and 0m respectively. The length of the ship is 105m and the depth is 9m. Calculate the following:
   a) the area of the waterplane.
   b) the transverse metacentric height at 7m draft if KG is 5m, KB is 4.5m and the block coefficient is 0.85. Is the ship stable ?

3. The half breadths of a waterplane of a ship 60m long are:

St No	0	0.5	1	2	3	4	5	6	7	8	9	9.5	10
½ B(m	2	3.1	4	5.3	5.9	6	6	6	5.1	3.8	2	1.2	0

   Calculate:
   a) the area of the waterplane
   b) the position of L.C.F from amidships
   c) the 2nd moment of area of waterplane about the longitudinal centreline of the ship.

4. A barge of constant waterplane area for all drafts has the following half breadths.

St.No	1	2	3	4	5	5.5	6
½.B(m	8	8	8	8	8	7.2	0

   The length of the barge is 50m and the depth is 4m. Calculate at 3.5m draft the transverse metacentric height assuming KB is 0.5 draft and KG is 6m. Is the barge stable at this condition?

5. The half breadths at 7.2m draft of a ship's waterplane from station 1 to 9 of total waterline length of 102m are as follows:

St.No.	1	3	5	7	9
½.B(m)	0	6.4	9.0	7.0	0

   Calculate the transverse GM at this draft if KB=4.3m, KG=5.3m and block coefficient = 0.78. Is the ship stable in this condition ?

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NAVAL ARCHITECTURE I
Tutorial No.10

1. The half ordinates of a ship at 7m draft starting from the aft are 0.2, 7.4, 8.7, 9.0, 9.1, 9.2, 9.1, 8.6, 7.8 , 5.1 and 0m respectively.The length of the ship is 105m and the depth is 9m. Calculate the longitudinal metacentric height at 7m draft if KG is 5m, KB is 4.5m and the block coefficient is 0.85

2. The half breadths of a waterplane of a ship 60m long are:

St No	0	.5	1	2	3	4	5	6	7	8	9	9.5	10
½ B	2	3.1	4	5.3	5.9	6	6	5	5.1	3.8	2	1.2	0

Calculate the longitudinal second moment of area of waterplane at an axis passing through the LCF.

3. A barge of constant waterplane area for all drafts has the following half breadths.

St.No	1	2	3	4	5	5.5	6
½B(m)	8	8	8	8	8	7.2	0

The length of the barge is 50m and the depth is 4m. Calculate at 3.5m draft the longitudinal metacentric height assuming KB is 0.5 draft and KG is 6m.

4. The half breadths at 7.2m draft of a ship's waterplane from station 0 to 9 of total waterline length of 102m, are as follows:

St.No.	1	3	5	7	9
½.B(m)	0	6.4	9.0	7.0	0

Calculate the longitudinal GM at this draft if KB = 4.3m, KG = 5.3m and the block coefficient = 0.78.

5. Using the theorem of parallel axis show that the transverse second moment of area of waterplane through the centreline of the waterplane is:

$$I_T = \tfrac{2}{3}\int_0^L y^3.dx$$

6. The half breadths at 5m draft of a barge 100m LBP are:

St.No	AP	2	4	6	8	9	FP
½.B(m	10	15	15	15	15	12	0

At this draft KG = 6m, KB = 2.5m and block coefficient = 0.92. Calculate :

a) the transverse GM
b) the longitudinal GM